Tuesday, March 16, 2021 1:43 PM

Indeterminate forms

$$\lim_{x \to 0} \frac{x^2 + 2x}{x+1} = ?$$

Put $x = 0$

$$= \frac{0+0}{0+1} = 0 \rightarrow \text{finite} \rightarrow \frac{0}{0}$$

While solving limits, we can have forms

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty^0, 0^0, 1^\infty, \infty - \infty$$

are called as indeterminate forms

L'Hospital's Rule for $\frac{0}{0}, \frac{\infty}{\infty}$

$$\lim_{x \to c} \frac{F(x)}{g(x)} = \lim_{x \to c} \frac{F'(x)}{g'(x)} = \lim_{x \to c} \frac{F''(x)}{g''(x)}$$

Que: $\lim_{x \to 0} \frac{\sin x}{x} = ?$

Put $x=0$, $\frac{0}{0}$ form, Apply L'Hospital Rule

Put $x=0$, $\frac{0}{0}$ form, Apply L Hospital Rule

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = \cos 0 = 1$$

Que $\lim_{x\to 0} \frac{\tan x}{x} = ?$ $\left(\frac{0}{0} \text{ form}\right)$

By L Hos Rule

$$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sec^2 x}{1} = 1$$

Que $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = ?$

$\frac{0}{0}$ form, By L-H Rule

$$\lim_{x\to 0} \frac{\cos ax \cdot a}{\cos bx \cdot b} = \frac{a}{b}$$

Que $\lim_{x\to 0} \frac{(x - \tan x)}{x - \sin x}$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x\to 0} \frac{(1 - \sec^2 x)}{1 - \cos x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 0} \frac{0 - 2\sec x\,(\sec x \tan x)}{} = \lim \left[-2\sec^2 x \tan x \right.$$

$$= \lim_{x\to 0} \frac{0 - 2\sec x(\sec x \tan x)}{0-(-\sin x)} = \lim_{x\to 0}\left[\frac{-2\sec^2 x \tan x}{\sin x}\right]$$

$$= \lim_{x\to 0} \frac{-2\,\frac{1}{\cos^2 x}\,\frac{\sin x}{\cos x}}{\sin x} = \lim_{x\to 0} \underbrace{-2\sec^3 x}$$

$$= -2$$

$\boxed{\text{OR}}$ $$\lim_{x\to 0} \frac{-2\sec^2 x \left(\frac{\tan x}{x}\right) x}{\sin x} \longleftarrow$$ only when fun are in mul

$$= \lim_{x\to 0} -2\sec^2 x = -2$$

Que $$\lim_{x\to 0} \frac{x e^x - \log(1+x)}{x^2}$$ $\left(\frac{0}{0}\text{ form}\right)$ $\log 1 = 0$ Use L'Hos Rule

Sol: $$\lim_{x\to 0} \frac{\underbrace{x e^x} + e^x - \frac{1}{1+x}}{2x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0} \frac{x e^x + e^x + e^x + \frac{1}{(1+x)^2}}{2} = \frac{2+1}{2} = \frac{3}{2}$$

Que Solve $\lim$ $e^x - e^{-x}$ ...

Qu: Solve $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$

$x \frac{\sin x}{x} x$

Sol: $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x^2}$ $\left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$

$\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \frac{e^x + e^{-x} - \frac{2}{1+x}}{2x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \frac{e^x - e^{-x} + \frac{2}{(1+x)^2}}{2} = \frac{2}{2} = 1$

Qu: $\lim_{x\to 0} \frac{\log x^2}{\cot x^2}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$\log 0 = -\infty$

$\cot 0 = \frac{1}{0} = \infty$

By L'Hospital Rule

$\lim_{x\to 0} \frac{\frac{1}{x^2}(2x)}{-\operatorname{cosec}^2 x^2 (2x)} = \lim_{x\to 0} \frac{-1/x^2}{\operatorname{cosec}^2 x^2}$

$= -\lim_{x\to 0} \frac{\sin^2 x^2}{x^2} \times \frac{x^2}{x^2}$

$x \to 0 \quad x^2 \quad x^2$

$$= -\lim_{x \to 0} \left(\frac{\sin^2 x^2}{x^4} \right) x^2$$

$$= -\lim_{x \to 0} \left(\frac{\sin x^2}{x^2} \right)^2 \cdot x^2$$

$$= -\lim_{x \to 0} x^2 \qquad \left[\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= 0$$

$$\underbrace{\frac{0}{0}, \frac{\infty}{\infty}}\Big] \rightarrow L'\text{Hospital Rule}$$

$$\lim_{x \to c} \frac{F(x)}{g(x)} = \lim_{x \to c} \frac{F'(x)}{g'(x)} = \cdots$$

Que: $\lim_{x \to 0} \frac{\log x^2}{\cot x^2} \rightarrow ?$

$\log 0 = -\infty$

$\cot 0 = \infty$

$\frac{\infty}{\infty}$ form, By L-H Rule,

$$\lim_{x \to 0} \frac{\frac{1}{x^2} \cdot \cancel{2x}}{-\operatorname{cosec}^2 x^2 \, \cancel{2x}} = \lim_{x \to 0} \left[-\frac{\sin^2 x^2}{x^2} \right]$$

$$= -\lim_{x \to 0} \left[\frac{\sin x^2}{x^2} \right] \underbrace{\sin x^2}$$

$$\boxed{\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1}$$

$$= 1 \cdot \sin 0 = 0$$

Que: $\lim_{x \to 0} \frac{x - \tan x}{x^3} = ?$

Put $x = 0$

$\frac{0 - \tan 0}{} = \frac{0}{}$

$x \to 0 \qquad x^2$

$\frac{0}{0}$ form, By L-H Rule

$$\lim_{x\to 0} \left(\frac{1-\sec^2 x}{3x^2}\right) = \lim_{x\to 0} \frac{-\tan^2 x}{3x^2}$$

$$= \frac{-1}{3} \lim_{x\to 0}\left[\frac{\tan x}{x}\right]^2$$

$$= \frac{-1}{3}(1) = \frac{-1}{3}$$

$\frac{0-\tan 0}{0} = \frac{0}{0}$

Recall

$\sec^2\theta - 1 = \tan^2\theta$

$\lim_{x\to 0} \frac{\tan x}{x} = 1$

II[nd] Method

$$\lim_{x\to 0} \frac{1-\sec^2 x}{3x^2} \cdot \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x\to 0} \frac{0 - 2\sec x(\sec x \tan x)}{6x}$$

$$\lim_{x\to 0} \frac{\frac{-1}{3}\frac{1}{\cos^2 x}\frac{\sin x}{\cos x}}{x}$$

$$= \lim_{x\to 0} \frac{-1}{3} \left(\frac{\sin x}{x}\right) \frac{1}{\cos^3 x} = \frac{-1}{3}$$

Recall

$\log_a b = \dfrac{\log b}{\log a}$

$\log 0 = -\infty$

Que: $\lim_{x \to 0} \log_{\sin 2x}(\sin x)$

Sol: $\lim_{x \to 0} \dfrac{\log(\sin x)}{\log(\sin 2x)}$ $\left(\dfrac{\infty}{\infty} \text{ form}\right)$

By L-H Rule

$$\lim_{x \to 0} \frac{\frac{1}{\sin x} \cos x}{\frac{1}{\sin 2x} \cos 2x \cdot 2} = \lim_{x \to 0} \frac{1}{2} \frac{\sin 2x \cos x}{\sin x \cos 2x}$$

$$= \lim_{x \to 0} \frac{1}{\cancel{2}} \frac{(\cancel{2} \cancel{\sin x} \cos x) \cos x}{\cancel{\sin x} \cos 2x}$$

$$= \frac{\cos^2 0}{\cos 0} = 1$$

Imp

Que: $\lim_{x \to \infty} \dfrac{x^n}{e^x} = ?$ where $n \in N$

Put $x = \infty$

$\dfrac{(\infty)^n}{e^\infty} = \dfrac{\infty}{\infty}$

$\dfrac{\infty}{\infty}$ form, By L-H Rule

Ist Time → $\lim_{x \to \infty} \dfrac{n x^{n-1}}{e^x}$

$\dfrac{\infty}{\infty}$ form, By L-H Rule

2nd Time → $\lim_{x \to \infty} \dfrac{n(n-1) x^{n-2}}{e^x}$ $\left(\dfrac{\infty}{\infty} \text{ form}\right)$

2^{nd} Time $\to \underset{x\to\infty}{lt} \dfrac{n(n-1)x}{e^x}$ $\left(\dfrac{\infty}{\infty} \text{ form}\right)$

After applying L-H Rule 'n' times, we get

$$\underset{x\to\infty}{lt} \frac{[n(n-1)(n-2)\cdots 3\cdot 2\cdot 1]\,x^{n-n}}{e^x}$$

$$\underset{x\to\infty}{lt} \frac{n!\,x^0}{e^x} = n!\,\underset{x\to\infty}{lt}\frac{1}{e^x} = n!\,\frac{1}{e^\infty}$$

$$= n!\left[\frac{1}{\infty}\right] = n!\,(0) = 0$$

Que. $\underset{x\to\infty}{lt} \dfrac{x^n}{e^x}$, $n \in N$ equals to

a) 1

b) 0

c) 1/2

d) one

$n = 1 \in N$

$$\underset{x\to\infty}{lt} \frac{x}{e^x} = \underset{x\to\infty}{lt} \frac{1}{e^x} = \frac{1}{\infty} = 0$$

Que. $\underset{x\to\infty}{lt} \dfrac{x+\sin x}{x+\cos x} = ?$

Sol: $$\underset{x\to\infty}{lt} \frac{\cancel{x}\left[1+\frac{\sin x}{x}\right]}{\cancel{x}\left[1+\frac{\cos x}{x}\right]}$$

$$= \lim_{x\to\infty} \frac{\left[1+\frac{\sin x}{x}\right]}{\left[1+\frac{\cos x}{x}\right]}$$

$$= 1$$

$-1 \le \sin x \le 1$

$\lim_{x\to\infty} \frac{\text{finite}}{x} = 0$

If $\lim_{x\to c} F(x)\cdot g(x)$ and $\lim_{x\to c} g(x) = 0$, $F(x)$ is bounded

then $\lim_{x\to c} F(x)g(x) = 0$

$$\lim_{x\to\infty} \underbrace{(\sin x)}_{\text{bdd}} \frac{1}{x} = 0$$

Que: $\lim_{x \to \frac{\pi}{2}} \left[\frac{\tan x}{\log(\cos x)} \right]$

Sol: $\frac{\infty}{\infty}$ form, By L-H Rule,

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec^2 x}{\frac{1}{\cos x}(-\sin x)}$$

$$= \lim_{x \to \frac{\pi}{2}} -\frac{1}{\cos^{\cancel{2}} x} \frac{\cancel{\cos x}}{\sin x}$$

$$= \lim_{x \to \pi/2} \frac{-1}{\sin x \cos x}$$

$$= \frac{-1}{(1)(0)} = -\infty$$

Put $x = \frac{\pi}{2}$

$$\frac{\tan \pi/2}{\log\left(\cos\frac{\pi}{2}\right)}$$

$$= \frac{\infty}{\log(0)}$$

$$= \frac{\infty}{-\infty}$$

Friday, March 19, 2021 9:55 AM

① $\frac{0}{0}$, $\frac{\infty}{\infty}$] By L-Hospital Rule

② $0\cdot\infty$, $\infty-\infty$] How to solve ?

$0\cdot\infty$ form : First Reduce it to $\frac{0}{0}$ or $\frac{\infty}{\infty}$

& then L'Hospital Rule

Que: $\lim_{x\to 0} x\log x = ?$

Put $x=0$

$\log 0 = -\infty$

$0\cdot\infty$ form

$0\cdot\infty = \frac{0}{0}$

or $\frac{\infty}{\infty}$

$0\cdot\infty = \frac{0}{1/\infty} = \frac{0}{0}$

$0\cdot\infty = \frac{\infty}{1/0} = \frac{\infty}{\infty}$

Que. $\lim_{x\to 0} x^{(1)}(\log x)'$ ($0\cdot\infty$ form)

Sol: $\lim_{x\to 0} \frac{\log x}{1/x}$ $\left(\frac{\infty}{\infty}\right)$

By L-H Rule,

$\lim_{x\to 0} \frac{1/x}{-1/x^2} = \lim_{x\to 0} (-x) = 0$

NOTE: If log is present then keep it in numerator

$\lim_{x\to 0} \frac{x}{1/\log x}$ $\left(\frac{0}{0}\right)$

By L-H Rule,

$\lim_{x\to 0} \frac{1}{-\left(\frac{1}{\log x}\right)^2 \cdot \frac{1}{x}}$

$= \lim_{x\to 0} \frac{x}{-\left(\frac{1}{\log x}\right)^2}$

we are not getting answer?

Qu: $\lim_{x\to 1} (1-x) \tan \frac{\pi x}{2} = ?$

(0 · ∞ form)

Put $x = 1$

$\tan\frac{\pi}{2} = \infty$

$0 \cdot \infty$ form

Sol: $\lim_{x\to 1} \frac{1-x}{\cot \frac{\pi x}{2}}$ $\left(\frac{0}{0} \text{ form}\right)$

By L-H Rule

$= \lim_{x\to 1} \frac{\cancel{-}1}{\cancel{-}\operatorname{cosec}^2 \frac{\pi x}{2} \cdot \frac{\pi}{2}} = \frac{1}{\operatorname{cosec}^2 \frac{\pi}{2} \cdot \frac{\pi}{2}} = \frac{2}{\pi}$

Qu: $\lim_{x\to 0} x^m (\log x)^n$ where $m, n \in N = \{1, 2, \ldots\}$

Qu. $\lim_{x\to 0} x^m (\log x)^n$ where $m, n \in N - \{1,2,\ldots\}$

Sol: $0\cdot\infty$ form,

$$\lim_{x\to 0} \frac{(\log x)^n}{1/x^m} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule, $\left[\frac{1}{x^m} = x^{-m}\right.$

$$\lim_{x\to 0} \frac{n(\log x)^{n-1}\left(\frac{1}{x}\right)}{-m\,x^{-m-1}}$$

$$\text{I}^{st} = \lim_{x\to 0} \frac{n(\log x)^{n-1}}{(-m)\,x^{-m}} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule

$$\lim_{x\to 0} \frac{n\,(n-1)\,(\log x)^{n-2}\cdot\left(\frac{1}{x}\right)}{(-m)\,(-m)\,x^{-m-1}}$$

$$2^{nd} = \lim_{x\to 0} \frac{n(n-1)(\log x)^{n-2}}{(-m)^2\,x^{-m}} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

After applying L-H Rule, 'n' times,

$$\lim \frac{[n(n-1)(n-2)\text{-----}3\cdot 2\cdot 1](\log x)^{n-n}}{}$$

$$\lim_{x\to 0} \frac{[n(n-1)(n-2)\cdots 3\cdot 2\cdot 1](\log x)^{-}}{(-m)^n\, x^{-m}}$$

$$\lim_{x\to 0} \frac{n!\, x^m}{(-m)^n} = 0$$

Qu: $\lim_{x\to 0} x^m (\log x)^n$ ✓Put $m=n=1$

$\lim_{x\to 0} x \log x = 0$

$(\infty - \infty)$ form

$$\lim_{x\to c}\left[f(x) - g(x)\right] = \boxed{\infty - \infty}$$

How to convert it in $\left(\frac{0}{0}\right)$ form.

$$f(x) - g(x) = \frac{1}{1/f(x)} - \frac{1}{1/g(x)}$$

a b

$$\lim_{x\to c}\left[f(x) - g(x)\right] = \frac{1/g(x) - 1/f(x)}{\frac{1}{f(x)}\cdot\frac{1}{g(x)}} \quad \left(\frac{0}{0}\right) \text{ form.}$$

$f(x) \to \infty$, $g(x) \to \infty$

$\frac{1}{f(x)} \to 0$, $\frac{1}{g(x)} \to 0$

Que: $\lim_{x\to\infty} \underbrace{x}\tan^{-1}\left(\frac{2}{x}\right) = ?$

Put $x=\infty$

$\tan^{-1}\left(\frac{2}{\infty}\right)$

$= \tan^{-1}(0)$

$= 0$

$\infty\cdot 0$ form

Sol: $\infty\cdot 0$ form

$$\lim_{x\to\infty} \frac{\tan^{-1}\left(\frac{2}{x}\right)}{1/x} \quad \left(\frac{0}{0}\text{ form}\right)$$

By L-H Rule

$$\lim_{x\to\infty} \frac{\frac{1}{1+\left(\frac{2}{x}\right)^2}\left[\frac{-2}{\cancel{x^2}}\right]}{-1/\cancel{x^2}}$$

$$= \lim_{x\to\infty} 2\left[\frac{x^2+4-4}{x^2+4}\right] = 2\left[1-\frac{4}{x^2+4}\right]$$

$$= 2\left[1-\frac{1}{\infty}\right]$$

$$= 2[1-0]$$

$$= 2$$

Que: $\lim_{x\to 0}\left[\underbrace{\operatorname{cosec}x}_{f(x)} - \underbrace{\frac{1}{x}}_{g(x)}\right]$

$$\lim_{x\to c} f(x)-g(x) = \frac{\frac{1}{g(x)} - \frac{1}{f(x)}}{\frac{1}{f(x)\,g(x)}}$$

Sol: $$= \lim_{x\to 0} \frac{\frac{1}{1/x} - \frac{1}{\operatorname{cosec}x}}{\frac{1}{\quad}} = \lim_{x\to 0}\frac{x-\sin x}{x.(\sin x)}$$

$$\lim_{x\to 0} \frac{}{\frac{1}{1/x \, \operatorname{cosec} x}} = \lim_{x\to 0} \frac{x - \ldots}{x(\sin x)} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to 0} \frac{1-\cos x}{x\cos x + \sin x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 0} \frac{\sin x}{-x\sin x + \cos x + \cos x} = \frac{0}{0+1+1} = 0$$

Que: $\lim_{x\to \frac{\pi}{2}} (\sec x - \tan x) = ?$

Sol: $\infty - \infty$ form

$$\lim_{x\to \frac{\pi}{2}} \left[\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right] = \lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$$

$\frac{0}{0}$ form, By L-H Rule,

$$\lim_{x\to \frac{\pi}{2}} \frac{\cancel{-}\cos x}{\cancel{-}\sin x} = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1} = \boxed{0}$$

Que: $\lim \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right] = ?$

Que: $\lim_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{\sin^2 x}\right] = ?$

Sol: $\infty-\infty$ form

$$\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^2\left[\frac{\sin^2 x}{x^2}\right]x^2}$$

$\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right.$

$$=\lim_{x\to 0}\frac{\sin^2 x - x^2}{x^4} \quad \left(\frac{0}{0}\text{ form}\right)$$

By L-H Rule,

$$=\lim_{x\to 0}\frac{(2\sin x\cos x)-2x}{4x^3}$$

$$=\lim_{x\to 0}\frac{\sin 2x - 2x}{4x^3} \quad \left(\frac{0}{0}\text{ form}\right)$$

By L-H Rule

$$=\lim_{x\to 0}\frac{\cos 2x\,(2)-2}{12x^2}$$

$$=\frac{2}{12}\lim_{x\to 0}\frac{\cos 2x - 1}{x^2} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$=\frac{1}{6}\lim_{x\to 0}\frac{-\sin 2x\,(2)}{2x}$$

$$=\frac{-2}{6}\lim\left(\frac{\sin 2x}{2x}\right)=\frac{-1}{3}(1)=\frac{-1}{3}$$

$$= \frac{-\cancel{2}}{6} \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right) = \frac{-1}{3}(1) = \frac{-1}{3}$$

Qu: $\lim_{x \to 0} \left[\frac{1}{e^x - 1} - \frac{1}{x}\right]$

Sol: $\lim_{x \to 0} \left[\frac{x - e^x + 1}{x(e^x - 1)}\right]$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 0} \left[\frac{1 - e^x}{x(e^x) + (e^x - 1)}\right] \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \left[\frac{-e^x}{xe^x + e^x + e^x}\right] = \frac{-1}{2}$$

$0^0, 1^\infty, \infty^0$ forms

$\lim_{x \to c} [F(x)]^{g(x)}$ — ?

Let $y = [F(x)]^{g(x)}$

Take log

$\log y = \log [F(x)]^{g(x)}$

$\to \log y = g(x) \log F(x) = l \to$ $\boxed{0 \cdot \infty \text{ Form}}$

Solve it by converting it into $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$\boxed{\lim_{x \to c} y = e^{l}} \leftarrow$

$0^0 \to \log 0^0 = 0 \log 0 = 0 \cdot \infty$

$1^\infty \to \log 1^\infty = \infty \log 1 = 0 \cdot \infty$

$\infty^0 \to \log \infty^0 = 0 \cdot \infty$

Qu: $\lim_{x \to 0} x^x = ?$

Sol: 0^0 form

Let $y = x^x$

Take log on both sides

$\log y = \log x^x$

$\log y = x \log x$

Take limit on both sides

$\lim \log y = \lim x \log x$

Put $x = 0$

$x^x = 0^0$

$[\because \log a^b = b \log a$

Take limit on both sides

$$\lim_{x\to 0} \log y = \lim_{x\to 0} x\log x \quad [0\cdot\infty \text{ form}]$$

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\log x}{1/x} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{1/x}{-1/x^2} = \lim_{x\to 0} (-x) = 0$$

$$\lim_{x\to 0} \log y = 0$$

$$\Rightarrow \lim_{x\to 0} y = e^0 = 1$$

If $\log y = x$
then $y = e^x$

Que: $\lim_{x\to 1} (x)^{\frac{1}{x-1}} = ?$

Put $x = 1$

$(x)^{\frac{1}{x-1}} = (1)^{\frac{1}{0}}$

$= 1^\infty$

Sol: 1^∞ form

Let $y = (x)^{\frac{1}{x-1}}$

Take log $\Rightarrow \log y = \log(x)^{\frac{1}{x-1}}$

$$\log y = \frac{1}{x-1}\log x$$

Apply limit $x\to 1$

$$\lim_{x\to 1} \log y = \lim_{x\to 1} \frac{\log x}{x-1} \quad \left(\frac{0}{0} \text{ form}\right)$$

Apply L-H Rule,

$$\lim_{x\to 1} \log y = \lim_{x\to 1} \frac{1/x}{1} = \lim_{x\to 1} \frac{1}{x} = 1$$

$$\Rightarrow \lim_{x\to 1} \log y = 1 \quad \Rightarrow \lim_{x\to 1} y = e^{1} = e$$

Que: $\lim_{x\to 0} (\operatorname{cosec} x)^{\frac{1}{\log x}}$

Put $x=0$
$\operatorname{cosec} 0 = \infty$
$\log 0 = -\infty$
$\frac{1}{\log 0} = \frac{1}{-\infty} = 0$

Sol: ∞^0 form

Let $y = (\operatorname{cosec} x)^{\frac{1}{\log x}}$

Take log

$$\log y = \frac{1}{\log x} \log \operatorname{cosec} x$$

Take limit $x \to 0$

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\log \operatorname{cosec} x}{\log x} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

$\boxed{\log \infty = \infty}$

By L-H Rule,

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\frac{1}{\operatorname{co\cancel{sec}x}} \cdot -\operatorname{co\cancel{sec}x} \cot x}{1/x}$$

$$\lim_{x\to 0} \log y = -\lim_{x\to 0} x \underbrace{\cot x} = -\lim_{x\to 0} \frac{x}{\tan x} = -1$$

$$\Rightarrow \lim_{x\to 0} \log y = -1$$

$$\Rightarrow \lim_{x\to 0} y = e^{-1} = \frac{1}{e}$$

$$\Rightarrow \lim_{x\to 0} y = e^{\cdot} = \frac{1}{e}$$

$$0^{\infty} - ?$$

$$\log 0^{\infty} = \infty \log 0 = \infty(-\infty)$$
$$= \underbrace{\infty}$$

Que: Prove $\lim_{x\to\infty} (1+x)^{\frac{1}{x}} = 1$

∞^0 form [Put $x = \infty$

Sol: ∞^0 form

Take $y = (1+x)^{\frac{1}{x}}$

Put $x = \infty$

$(1+\infty)^{\frac{1}{\infty}} = (\infty)^0$

Take log, $\log y = \frac{1}{x}\log(1+x)$

Take limit $x \to \infty$

$$\lim_{x\to\infty} \log y = \lim_{x\to\infty} \frac{\log(1+x)}{x} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to\infty} \log y = \lim_{x\to\infty} \frac{\frac{1}{1+x}}{1}$$

$$= \lim_{x\to\infty} \frac{1}{1+x} = \frac{1}{1+\infty}$$

$$\lim_{x\to\infty} \log y = \frac{1}{\infty} = 0$$

$$\Rightarrow \lim_{x\to\infty} \log y = 0$$

$$\Rightarrow \lim_{x\to\infty} y = e^0 = 1$$

Que: Prove $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

Que: $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} = ?$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$(1)^\infty$ form

$\lim \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

$x \to 0 \left(\quad x \quad \right)$. | (1) form

$$y = \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$$

$$\log y = \frac{1}{x^2} \log\left(\frac{\sin x}{x}\right)$$

Limit $x \to 0$, $\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\log\left(\frac{\sin x}{x}\right)}{x^2}$ $\left(\frac{0}{0}\right.$ form$\left.\right)$

By L-H Rule

$$= \lim_{x\to 0} \frac{\frac{1}{\left(\frac{\sin x}{x}\right)}\left[\frac{x\cos x - \sin x}{x^2}\right]}{2x}$$

$$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{x\cos x - \sin x}{2x^3} \quad \left[\frac{0}{0} \text{ form}\right]$$

By L-H Rule

$$= \lim_{x\to 0} \frac{-x\sin x + \cos x - \cos x}{6x^2}$$

$$= \lim_{x\to 0} \frac{-x\sin x}{6x^2} = \lim_{x\to 0} \frac{-1}{6} \frac{\sin x}{x}$$

$$\lim_{x\to 0} \log y = \frac{-1}{6}\left[\lim_{x\to 0} \frac{\sin x}{x}\right] = \frac{-1}{6}$$

$$\Rightarrow \lim_{x\to 0} y = e^{-1/6}$$

Tuesday, March 23, 2021 1:40 PM

① $\frac{0}{0}$, $\frac{\infty}{\infty}$

② $0\cdot\infty$, $\infty-\infty$

③ 0^0, 1^∞, ∞^0

Que: If $\lim_{x\to 0} \frac{\sin 2x + a\sin x}{x^3}$ be finite, find value of a & limit also

Sol:

$$\lim_{x\to 0} \frac{\sin 2x + a\sin x}{x^3} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to 0} \frac{\cos 2x\,(2) + a\cos x}{3x^2} \quad \text{——— ①}$$

When $x=0$, $\frac{\cos 0\,(2) + a\cos 0}{3(0)} = \frac{2+a}{0} \left(\frac{0}{0}\right)$

$\because$ Limit is finite, so $2+a=0$

$$\boxed{a=-2}$$

Put $a=-2$ in ①

$$\lim_{x\to 0} \frac{2\cos 2x - 2\cos x}{3x^2} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to 0} \frac{-4\sin 2x + 2\sin x}{6x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x\to 0} \frac{-8\cos 2x + 2\cos x}{6}$$

$$= \frac{-8+2}{6} = \frac{-6}{6} = -1$$

Que: Given $\lim_{x\to 0} \frac{\sin x + ax + bx^3}{x^5}$ is finite, find a & b.

$x \to 0 \qquad x^5$

Find a & b.

Sol: $\displaystyle \lim_{x \to 0} \frac{\sin x + ax + bx^3}{x^5}$ $\left(\frac{0}{0} \text{ form}\right)$

By L-H Rule,

$$\lim_{x \to 0} \frac{\cos x + a + 3bx^2}{5x^4} \quad \text{—— (1)}$$

When $x = 0$, $\dfrac{\cos 0 + a + 3b(0)}{0} = \dfrac{1+a}{0}$

$\because$ Given Limit is finite, $1 + a = 0$

$\Rightarrow \boxed{a = -1}$

Put $a = -1$ in (1)

$$\lim_{x \to 0} \frac{\cos x - 1 + 3bx^2}{5x^4} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule,

$$\lim_{x \to 0} \frac{-\sin x - 0 + 6bx}{20x^3} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule, $\displaystyle \lim_{x \to 0} \frac{-\cos x + 6b}{60x^2}$

When $x = 0$, $\dfrac{-\cos 0 + 6b}{60(0)} = \dfrac{-1+6b}{0}$

$\because$ Limit is finite $\Rightarrow$ $-1 + 6b = 0$

$$\boxed{b = \frac{1}{6}}$$

Qu: Find value of a & b so that $\displaystyle \lim_{x \to 0} \frac{x(1 - a\cos x) + b\sin x}{x^3}$ exists & equals to $\frac{1}{3}$

Sol: $\displaystyle \lim_{x \to 0} \frac{x(1 - a\cos x) + b\sin x}{x^3}$ $\left(\frac{0}{0} \text{ form}\right)$

By L-H Rule,

$$\lim_{x\to0} \frac{x(a\sin x)+(1-a\cos x)+b\cos x}{3x^2} \quad \text{——} (*)$$

When $x=0$, $\dfrac{0+1-a\cos0+b\cos0}{3(0)}$

$$= \frac{1-a+b}{0}$$

$\because$ Limit is finite, So, $\boxed{1-a+b=0}$

$\boxed{a=1+b}$ —— (1)

Put $a=1+b$ in $(*)$

$$\lim_{x\to0} \frac{x(1+b)\sin x+1-(1+b)\cos x+b\cos x}{3x^2}$$

$$\lim_{x\to0} \frac{(1+b)\,x\sin x+1-\cos x-\cancel{b\cos x}+\cancel{b\cos x}}{3x^2}$$

$$= \lim_{x\to0} \frac{(1+b)\,x\sin x+1-\cos x}{3x^2} \quad \left(\frac{0}{0}\text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to0} \frac{(1+b)\left[x\cos x+\sin x\right]+0+\sin x}{6x} \quad \left(\frac{0}{0}\text{ form}\right)$$

By L-H Rule,

$$\lim_{x\to0} \frac{(1+b)\left[(-x\sin x+\cos x)+\cos x\right]+\cos x}{6}$$

$$= \frac{(1+b)\left[0+1+1\right]+1}{6} = \frac{2(1+b)+1}{6}$$

$$= \frac{2b+3}{6}$$

A.T.Q, limit $= \dfrac{1}{3}$ ie $\boxed{\dfrac{2b+3}{6}=\dfrac{1}{3}}$

$2b+3=2 \Rightarrow \boxed{b=\dfrac{-1}{2}}$

From (1), $a=1+b$

From ①, $a = 1 + b$

$$= 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \boxed{a = \frac{1}{2}}$$

Que: find value of a, b, c if

$$\lim_{x \to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x\sin x} = 2$$

Sol:

$$\lim_{x \to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x\sin x} \quad \text{———} \; (*)$$

When $x = 0$, $\frac{a - b + c}{0}$

$\because$ Limit is 2 (finite) so, $a - b + c = 0$

$$\boxed{a + c = b} \quad \text{——} \; ①$$

Put $b = a + c$ in $(*)$

$$\lim_{x \to 0} \frac{ae^{x} - (a+c)\cos x + ce^{-x}}{x\sin x} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule

$$\lim_{x \to 0} \frac{ae^{x} + (a+c)\sin x - ce^{-x}}{x\cos x + \sin x} \quad \text{——} \; (**)$$

when $x = 0$, $\frac{a + 0 - c}{0} = \frac{a - c}{0}$

$\because$ Limit is finite, so, $a - c = 0$

$$\boxed{a = c} \quad \text{——} \; ②$$

Put $a = c$ in $(**)$

$$\lim_{x \to 0} \frac{ae^{x} + 2a\sin x - ae^{-x}}{x\cos x + \sin x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x \to 0} \frac{ae^{x} + 2a\cos x + ae^{-x}}{-x\sin x + \cos x + \cos x}$$

$$= \frac{a + 2a + a}{\ldots} = \frac{4a}{2} = 2a$$

$$= \frac{a+2a+a}{0+1+1} = \frac{4a}{2} = 2a$$

Also, Given limit = 2

$$\Rightarrow \quad 2a = 2 \Rightarrow \boxed{a=1}$$

From (2), $a = c \Rightarrow \boxed{c=1}$

From (1), $b = a+c \Rightarrow \boxed{b=2}$

Qu: $\lim_{x\to 0} \frac{\left(\cot x - \frac{1}{x}\right)}{x} \longrightarrow$ Simplify

Sol: $\lim_{x\to 0} \frac{\left(\frac{1}{\tan x} - \frac{1}{x}\right)}{x} = \lim_{x\to 0} \frac{x - \tan x}{x^3 \frac{\tan x}{x}}$

As we know, $\lim_{x\to 0} \frac{\tan x}{x} = 1$

$$= \lim_{x\to 0} \frac{x - \tan x}{x^3} \quad \left(\frac{0}{0} \text{ form}\right)$$

By L-H Rule $= \lim_{x\to 0} \frac{1-\sec^2 x}{3x^2} = \lim_{x\to 0} \frac{-\tan^2 x}{3x^2} = \frac{-1}{3} \lim_{x\to 0} \left(\frac{\tan x}{x}\right)^2$

$$= \frac{-1}{3}(1)^2 = \frac{-1}{3}$$

Qu: $\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = ?$

Put $x = \infty$

$e^{\infty} = \infty$

$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty}$

$e^{-\infty} = 0$

$\frac{\infty}{\infty}$ form, By L-H Rule

$$= \lim_{x\to\infty} \frac{e^x - e^{-x}(-1)}{e^x + e^{-x}(-1)} = \lim_{x\to\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Again we'll have $\frac{\infty}{\infty}$ form

$$\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x\to\infty} \frac{e^x - \frac{1}{e^x}}{\ldots}$$

$$\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x+e^{-x}} = \lim_{x\to\infty} \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}}$$

$$= \lim_{x\to\infty} \frac{\frac{e^{2x}-1}{\cancel{e^x}}}{\frac{e^{2x}+1}{\cancel{e^x}}} = \lim_{x\to\infty} \frac{e^{2x}-1}{e^{2x}+1} \left(\frac{\infty}{\infty} \text{ form}\right)$$

By L-H Rule, $\lim_{x\to\infty} \frac{e^{2x}(2)-0}{e^{2x}(2)+0} = \lim_{x\to\infty} \frac{2e^{2x}}{2e^{2x}}$

$$= 1$$

Qu: Evaluate $\lim_{x\to 0}\left[\cot^2 x - \frac{1}{x^2}\right]$

Put $x=0$

$\cot 0 = \frac{\cos 0}{\sin 0} = \frac{1}{0} = \infty$

Sol: $\infty - \infty$ form

$$\lim_{x\to 0}\left[\frac{1}{\tan^2 x} - \frac{1}{x^2}\right]$$

$$= \lim_{x\to 0} \frac{x^2 - \tan^2 x}{x^2\left[\frac{\tan^2 x}{x^2}\right]x^2} = \lim_{x\to 0} \frac{x^2-\tan^2 x}{x^4} \left(\frac{0}{0}\text{ form}\right)$$

$$\left[\because \lim_{x\to 0}\frac{\tan x}{x} = 1\right]$$

By L-H Rule,

$$\lim_{x\to 0} \frac{2x - 2\tan x \sec^2 x}{4x^3} = \lim_{x\to 0}\frac{x - \tan x \sec^2 x}{2x^3}$$

$$= \lim_{x\to 0}\frac{x - \tan x(1+\tan^2 x)}{2x^3} = \lim_{x\to 0}\frac{x - \tan x - \tan^3 x}{2x^3}$$

$\frac{0}{0}$ form, By L-H Rule

$$= \lim_{x\to 0} \frac{(1-\sec^2 x) - 3\tan^2 x\cdot\sec^2 x}{6x^2}$$

$$= \lim_{x\to 0}\frac{-\tan^2 x - 3\tan^2 x \sec^2 x}{6x^2}$$

$$= \lim_{x\to 0}\frac{\tan^2 x}{x^2}\left[\frac{-1-3\sec^2 x}{6}\right]$$

$$= \lim_{x \to 0} \frac{\tan x}{x^2} \left[\frac{-1 - 3\sec x}{6} \right]$$

$$= (1) \left[\frac{-1-3}{6} \right] = \frac{-4}{6} = \frac{-2}{3}$$

Tuesday, March 30, 2021 2:44 PM

Qu: $\lim_{x \to 0} \frac{1}{\sin^2 x} = ? \rightarrow$? undefined

Qu: $\lim_{p \to \infty} \frac{p^5 \, p!}{5 \cdot 6 \cdot 7 \cdots (5+p)}$

Sol: $\lim_{p \to \infty} \frac{p^5 \, p!}{(p+5)(p+4) \cdots 6 \cdot 5}$

$\lim_{p \to \infty} \frac{p^5 \, p! \, (4 \cdot 3 \cdot 2 \cdot 1)}{(p+5)(p+4) \cdots 6 \cdot 5 (4 \cdot 3 \cdot 2 \cdot 1)}$

$= \lim_{p \to \infty} \frac{p^5 \, p! \, 4!}{(p+5)!}$

$= \lim_{p \to \infty} \frac{p^5 \, 4! \, \cancel{p!}}{(p+5)(p+4)(p+3)(p+2)(p+1)\cancel{p!}}$

$= \lim_{p \to \infty} \frac{p^5 \, 4!}{(p+5)(p+4)(p+3)(p+2)(p+1)}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim_{p \to \infty} \frac{\cancel{p^5} \, 4!}{\cancel{p^5}\left[1+\frac{5}{p}\right]\left[1+\frac{4}{p}\right]\left[1+\frac{3}{p}\right]\left[1+\frac{2}{p}\right]\left[1+\frac{1}{p}\right]}$

$= 4!$

Qu: ... x^2+3x+1

Sol: $\lim_{x\to\infty}\left[1+\frac{1}{x^2+2x+1}\right]$

Sol: Let $y=\left[1+\frac{1}{x^2+2x+1}\right]^{x^2+3x+1}$

Take log $\log y=(x^2+3x+1)\log\left[1+\frac{1}{x^2+2x+1}\right]$

Take lt $x\to\infty$

$$\lim_{x\to\infty}\log y=\lim_{x\to\infty}(x^2+3x+1)\log\left[1+\frac{1}{x^2+2x+1}\right]$$

$$=\lim_{x\to\infty}\frac{\log\left[1+\frac{1}{x^2+2x+1}\right]}{1/x^2+3x+1}$$

$\frac{0}{0}$ form, By L-H Rule,

$$=\lim_{x\to\infty}\frac{\frac{1}{\left[1+\frac{1}{x^2+2x+1}\right]}\left(0-\frac{(2x+2)}{(x^2+2x+1)^2}\right)}{-\frac{(2x+3)}{(x^2+3x+1)^2}}$$

$$\lim_{x\to\infty}\frac{1}{\left[1+\frac{1}{x^2+2x+1}\right]}\frac{2x+2}{2x+3}\left[\frac{x^2+3x+1}{x^2+2x+1}\right]^2$$

$$\lim_{x\to\infty}\frac{1}{1+\frac{1}{x^2+2x+1}}\frac{\cancel{x}\left[2+\frac{2}{x}\right]}{\cancel{x}\left[2+\frac{3}{x}\right]}\left[\frac{\cancel{x^2}\left[1+\frac{3}{x}+\frac{1}{x^2}\right]}{\cancel{x^2}\left[1+\frac{2}{x}+\frac{1}{x^2}\right]}\right]^2$$

$$=\frac{1}{1+\frac{1}{\infty}}\left[\frac{2+\frac{2}{\infty}}{2+\frac{3}{\infty}}\right]\left[\frac{1+\frac{3}{\infty}+\frac{1}{\infty}}{1+\frac{2}{\infty}+\frac{1}{\infty}}\right]^2$$

$$\lim_{x\to\infty} \log y = \left[\frac{2}{2}\right]\left[\frac{1}{1}\right] = 1$$

$$\lim_{x\to\infty} y = e^1 = \boxed{e}$$

II Method

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\lim_{x\to 0} \frac{2\cos 2x + 3\cos 5x + 5\cos 19x}{\cos 4x - \cos 3x}$$

$\frac{0}{0}$, L-H

$$\lim_{x\to 0} \frac{-4\sin 2x - 15\sin 5x - 95\sin 19x}{-4\sin 4x + 3\sin 3x}$$

$\frac{0}{0}$, L-H Rule,

$$\lim_{x\to 0} \frac{-8\cos 2x - 15(5)\cos 5x - 95(19)\cos 19x}{-16\cos 4x + 9\cos 3x}$$

$$= \frac{-8 - 75 - 19(95)}{-16+9} = \_\_\_$$

None